

- In the path integration representation of the ground state wavefunction for the 1-dimensional quantum rotor model, an instanton describes the tunneling from states with winding number ν to states with winding number $\nu + 1$, and an anti-instanton describes the tunnelings from winding numbers $\nu + 1$ to ν . Consider a spacetime path where an instanton and an anti-instanton are located at position (x_1, τ_1) and (x_2, τ_2) , respectively. Such a configuration contributes to the path integration that connects two states with the same winding number (say $\nu = 0$).
 - Estimate the action of the spacetime configuration with an instanton and an anti-instanton.
 - Estimate the contribution of a pair of instanton and anti-instanton with $\sqrt{(x_1 - x_2)^2 + (\tau_1 - \tau_2)^2} < R$ with a fixed $(\frac{x_1+x_2}{2}, \frac{\tau_1+\tau_2}{2})$. Show that there exists a critical value of t/U below which the average distance between the instanton and anti-instanton pair diverges.

Further references :

- A pedagogical review on other emergent phenomena in condensed matter systems :
<https://arxiv.org/abs/1009.5127>