Consider the rotor model in the one-dimensional chain with L sites and the periodic boundary condition,

$$\hat{H} = \frac{U}{4}\sum_{j=1}^{L} \hat{n}_j^2 - 2t\sum_{j=1}^{L} \cos(\hat{\theta}_j - \hat{\theta}_{j+1}), \tag{1}$$

where $\hat{\theta}_{L+1} = \hat{\theta}_1$. $\hat{\theta}_j$ represents the angular variable defined at site j and $\hat{n}_j$ is the angular momentum operator that satisfies $[\hat{\theta}_j, \hat{n}_k] = i\delta_{jk}$. Let us write the (unnormalized) ground state in the limit that $t/U \ll 1$ as

$$\Psi_G(\theta_1, ..., \theta_L) = \prod_{i=1}^{L} \psi_0(\theta_i) + \sum_{i,j=1}^{L} c_{ij}\psi_1(\theta_i)\psi_{-1}(\theta_j)\left[\prod_{k\neq i,j} \psi_0(\theta_k)\right] + ..., \tag{2}$$

where $\psi_m(\theta) = \frac{1}{\sqrt{2\pi}}e^{im\theta}$. Show that c_{ij} decays as $\left(\frac{t}{U}\right)^{d_{ij}}$, where d_{ij} denotes the minimum number of links needed to connect site i and j.