1. Suppose $\hat{n}$ is the angular momentum operator of a rotor and $\hat{H}$ is Hamiltonian. Show that $[\hat{H}, \hat{n}] = 0$ implies

   (a) $\int d\theta \; \psi^*(\theta)\hat{H}\psi(\theta) = \int d\theta \; \psi'^*(\theta)\hat{H}\psi'(\theta)$, where $\psi'(\theta) = \psi(\theta - \delta)$.

   (b) $\int d\theta \; \psi^*(\theta, 0)\hat{n}\psi(\theta, 0) = \int d\theta \; \psi'^*(\theta, t)\hat{n}\psi'(\theta, t)$, where $\psi(\theta, t)$ is the state obtained by evolving the initial state $\psi(\theta, 0)$ for time t.

2. Let the total Hamiltonian of a single rotor is written as $\hat{H} = H_t(\hat{\theta}) + H_U(\hat{n})$, where H_t and H_U depend only on $\hat{\theta}$ and $\hat{n}$, respectively.

   (a) Show that $e^{-\epsilon \hat{H}} = e^{-\epsilon \hat{H}_U} e^{-\epsilon \hat{H}_t} + O(\epsilon^2)$

   (b) Show that

$$e^{-\epsilon \hat{H}_U} e^{-\epsilon \hat{H}_t} \Psi(\theta) = \sum_{n'} \int \frac{d\theta'}{2\pi} e^{in'(\theta - \theta') - \epsilon H_t(\theta') - \epsilon H_U(n')} \Psi(\theta'). \tag{1}$$

   (c) Show that

$$e^{-T\hat{H}} \Psi(\theta) = \lim_{\epsilon \to 0} \sum_{n^{(1)}, n^{(2)}, .., n^{(M)}} \int \frac{d\theta^{(1)}}{2\pi} \frac{d\theta^{(2)}}{2\pi} \cdots \frac{d\theta^{(M)}}{2\pi} e^{\sum_{l=1}^{M} \left[in^{(l)}(\theta^{(l+1)} - \theta^{(l)}) - \epsilon H_t(\theta^{(l)}) - \epsilon H_U(n^{(l)}) \right]} \Psi(\theta^{(1)}) \Bigg|_{M = T/\epsilon}, \tag{2}$$

   where $\theta^{(M+1)} = \theta$.