Effective Field Theory - EFT

1) Motivation

1.1) Meaning

An EFT is a field theory for describing the dynamics of a system at length scales large compared to a given cutoff.

Long wavelengths are effectively decoupled from short wavelengths

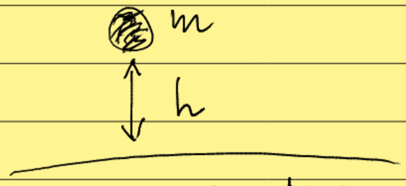
$\Updownarrow$

There is a natural separation of scales in the problem

Statements can be refrased in terms of energy scales $\rightarrow$ EFT, a theory for the dynamics at energies smaller compared to a given cutoff energy

Two examples in classical physics:

1.2) Gravity close to Earth's surface



$$F = mg \quad \text{------- (1)}$$

– get this from the "fundamental" Newton's law

M_E : Earth's mass

R_E : " radius

$$F = \frac{G m M}{(R_E + h)^2} \quad \text{-- (2)}$$

For $h \ll R_E$:

$$F = m \frac{GM}{R_E^2} \frac{1}{(1 + h/R_E)^2} \simeq m \underbrace{\frac{GM}{R_E^2}}_{g} \left(1 - 2\frac{h}{R_E} + 3\frac{h^2}{R_E^2} + \dots\right) \quad \text{---- (3)}$$

$h = 10\,m$, $R_E \simeq 6371\,Km$ ← equator

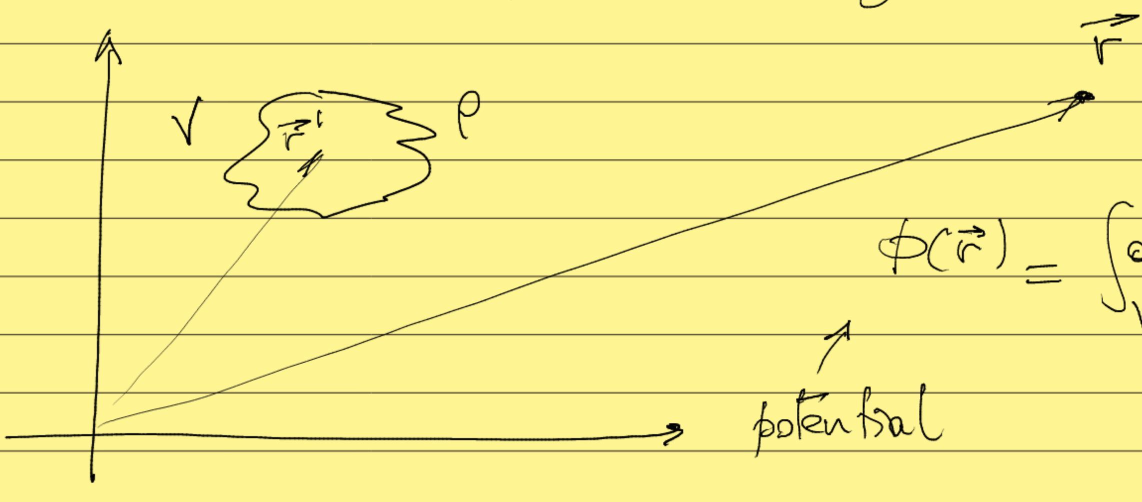
$$= mg\,(1 - 0.00000031 + 0.000\,000\,000\,000\,074 + \dots)$$

← when this level of precision is OK forget about Newton's law

Larger h : need Newton's law, e.g. for satelites

for GPS, Newton's law is not enough, need GR

1.3) Multipole expansion: charge distribution $\rho(\vec{r})$



$$\phi(\vec{r}) = \int_V d^3r' \frac{\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \quad \text{---- (4)}$$

↑ potential

– separation of scales $\langle r' \rangle = \int d^3r' \, r' \rho(\vec{r}\,')$

$$\langle r' \rangle \ll r \qquad \text{------- (5)}$$

– expansion of integrand in powers of $\langle r' \rangle / r$

power counting

$$\frac{1}{|\vec{r}-\vec{r}\,'|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} \frac{4\pi}{2l+1} \frac{r_<^{\,l}}{r_>^{\,l+1}} Y_{lm}^*(\theta', \varphi') Y_{lm}(\theta, \varphi)$$

$\vec{r}\,'$ $\vec{r}$

------ (6)

Using this in (4) for $r > r'$: $r_> = r$, $r_< = r'$

$$\phi(\vec{r}) = \sum_{l,m} \frac{4\pi}{2l+1} \frac{Y_{lm}(\theta,\varphi)}{r^{l+1}} Q_{lm} \qquad \text{------ (7)}$$

$$Q_{lm} = \int_V d^3r' \, \rho(\vec{r}\,') \, r'^{\,l} \, Y_{lm}^*(\theta', \varphi') \qquad \text{----- (8)}$$

$\Downarrow$

$$\phi(\vec{r}) = \frac{Q}{r} + \frac{d}{r^2} + \dots \qquad \text{------ (9)}$$

$$Q = \int_V d^3r \, \rho(r) \quad : \text{point charge} \qquad \text{---- (10)}$$

$$d = \frac{4\pi}{3} \sum_{m=-1}^{+1} Y_{1m}(\theta,\varphi) \int d^3r' \, r' \rho(\vec{r}\,') \, Y_{1m}^*(\theta',\varphi') \qquad \text{-- (11)}$$

$\hookrightarrow$ dipole

The closer one gets to the charge distribution, more structure one sees: $r \sim \langle r' \rangle$, breakdown scale

1.4) Aim of this course: at the end, you should be able to understand (and reproduce) the following example

QFT, two spinless particles, one is much lighter than the other: fields $\phi(x)$ and $\Phi(x)$ masses m and M, with $m \ll M$

$\boxed{\text{Kaplan, Burgess}}$

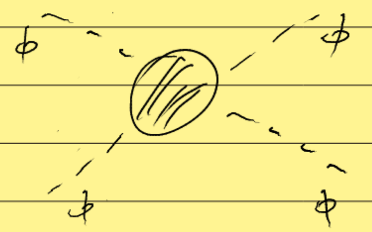
Classical action:

$$S_c = \int d^4x \left[\tfrac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + \tfrac{1}{2} \partial_\mu \Phi \, \partial^\mu \Phi - V(\phi, \Phi) \right] \quad \text{---(12)}$$

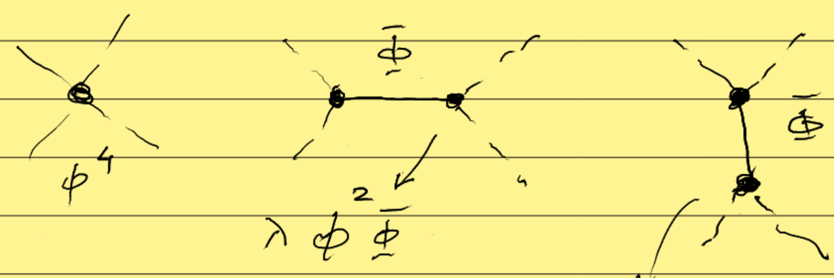
$$V(\phi, \Phi) = \tfrac{1}{2} m^2 \phi^2 + \tfrac{1}{2} M^2 \Phi^2 + \tfrac{1}{4!}\, g\, \phi^2 + \tfrac{1}{2} \lambda\, \phi^2 \Phi + \dots \quad \text{----(13)}$$

dimensionful

Consider ϕ-ϕ scattering with c.m. energies $E_{cm} \ll M$

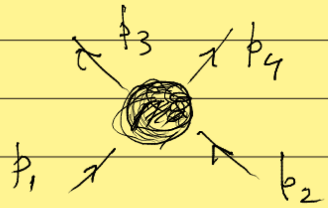


$\rightarrow$ lowest order in perturbation theory



S-matrix element:

$$S(p_1, p_2; p_3, p_4) = i\,(2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4)\, A(p_1, p_2, p_3, p_4) \quad \text{-----(14)}$$



$$A(p_1, p_2, p_3, p_4) = -g - \lambda^2 \left[\frac{1}{(p_1-p_3)^2 - M^2} + \frac{1}{(p_1-p_4)^2 - M^2} \right.$$

$$\left. + \frac{1}{(p_1+p_2) - M^2} \right] = -g + \lambda^2 \frac{1}{M^2} \left[\frac{1}{1 - (p_1-p_3^2)/M^2} \right.$$

$$\left. + \frac{1}{1 - (p_1-p_4)^2/M^2} + \frac{1}{1 - (p_1+p_2)^2/M^2} \right]$$

$$\cong -g + \frac{\lambda}{M^2} \left\{ 3 + \frac{1}{M^2} \left[\underbrace{(p_1-p_3)^2 + (p_1-p_4)^2 + (p_1+p_2)^2}_{} \right] \cdots \right\}$$

On-shell $p_i^2 = m^2$

$$3p_1^2 + p_3^2 + p_4^2 + p_2^2 - 2p_1 \cdot p_3 - 2p_1 \cdot p_4 + 2p_1 \cdot p_2$$

$$= 6m^2 + 2p_1 \cdot (\underbrace{p_2 - p_3 - p_4}_{\to -p_1})$$

$$= 6m^2 - 2p_1^2 = 4m^2$$

$$= -g^2 + \frac{3\lambda^2}{M^2} + \frac{4\lambda^2 m^2}{M^4} \quad \text{------- (15)}$$

Observations:

1) Calculation of A simplifies when one can neglect powers of m/M, λ/M, $E_{c.m}/M$

2) Eq. (15) can be obtained from a theory involving only the ℓ field:

$$V_{eff}^{(4)} = 1/4! \left(g - 3\lambda^2/M^2 - 4\lambda^2 m^2/M^4 \right) \phi^4$$

----- (16)

Knowing that same result is obtained when using (16) and (17) when $m/M \ll 1$:

$\Downarrow$

- makes life easier instead of expanding full result
- guidance to what observables at low energies are more sensitive to properties of heavy particles

Basic property of a QFT:

- when a large hierarchy of masses exists, a low energy description in terms of an EFT seems to be always possible.

1.5) How to construct an EFT?

↳ until Friday, you will know!

1.6) Categories of EFTs: { generically, two ways they are used

$\boxed{\text{top-down \& bottom-up}}$

Lagrangian is known

- **Top-down:** High-energy theory is known

But we would like to have a simpler theory at low energies

- "integrate out" heavy particles and

- match onto a low energy theory
  - ↳ find new operators & new low energy constants, e.g. previous example

$$\mathcal{L} \rightarrow \mathcal{L}_{EFT} = \sum_n \mathcal{L}^{(n)}_{low}$$ ← expansion in decreasing importance

describes more than, but they coincide in the infrared and differ in in the ultraviolet

— Bottom-up : high energy theory is not known, or it is known but matching cannot be done when integrating out degrees of freedom
↑ too difficult

* Construct $\sum_n \mathcal{L}^{(n)}_{Low}$ using most general operators / interactions consistent with symmetries

** Couplings are unknown ← determine from experiment

or → from full theory, use lattice ← QCD

**+ Precision desired → what n to stop

Example : 1) superconductivity ← see e.g. Kaplan

2) QCD : chiral perturbation theory

3) Einstein gravity → quantum and operators consistent with GCT