- For one complex variable $x = x_R + ix_I$, verify the following formula :

$$\frac{\int dx_R dx_I \; e^{-\Delta|x|^2 + i\alpha x + i\alpha x^*}}{\int dx_R dx_I \; e^{-\Delta|x|^2}} = e^{-\frac{\alpha^2}{\Delta}} \tag{1}$$

- From this, verify the expression marked as (Ex) in page 14 of the lecture note. (Note that $\theta_{\omega,-\vec{k}} = \theta^*_{\omega,\vec{k}}$ and $\sum_{\omega,\vec{k}}$ in the note denotes the sum over all allowed frequencies and momenta).