Consider the rotor model in the one-dimensional chain with $L$ sites and the periodic boundary condition,

$$
\hat{H} = \frac{U}{4} \sum_{j=1}^{L} \hat{n}_j^2 - 2t \sum_{j=1}^{L} \cos(\hat{\theta}_j - \hat{\theta}_{j+1}),
$$

(1)

where $\hat{\theta}_{L+1} = \hat{\theta}_1$. $\hat{\theta}_j$ represents the angular variable defined at site $j$ and $\hat{n}_j$ is the angular momentum operator that satisfies $[\hat{\theta}_j, \hat{n}_k] = i\delta_{jk}$. Let us write the (unnormalized) ground state in the limit that $t/U \ll 1$ as

$$
\Psi_G(\theta_1, ..., \theta_L) = \prod_{i=1}^{L} \psi_0(\theta_i) + \sum_{i,j=1}^{L} c_{ij} \psi_1(\theta_i) \psi_{-1}(\theta_j) \left[ \prod_{k \neq i,j} \psi_0(\theta_k) \right] + ..., 
$$

(2)

where $\psi_m(\theta) = \frac{1}{\sqrt{2\pi}} e^{im\theta}$. Show that $c_{ij}$ decays as $(\frac{t}{U})^{d_{ij}}$, where $d_{ij}$ denotes the minimum number of links needed to connect site $i$ and $j$. 