1. Suppose \( \hat{n} \) is the angular momentum operator of a rotor and \( \hat{H} \) is Hamiltonian. Show that \([\hat{H}, \hat{n}] = 0\) implies

(a) \[ \int d\theta \; \psi^*(\theta) \hat{H} \psi(\theta) = \int d\theta \; \psi'^*(\theta) \hat{H} \psi'(\theta), \]
where \( \psi'(\theta) = \psi(\theta - \delta) \).

(b) \[ \int d\theta \; \psi^*(\theta, 0) \hat{n} \psi(\theta, 0) = \int d\theta \; \psi'^*(\theta, t) \hat{n} \psi'(\theta, t), \]
where \( \psi(\theta, t) \) is the state obtained by evolving the initial state \( \psi(\theta, 0) \) for time \( t \).

2. Let the total Hamiltonian of a single rotor is written as \( \hat{H} = H_t(\hat{\theta}) + H_U(\hat{n}) \), where \( H_t \) and \( H_U \) depend only on \( \hat{\theta} \) and \( \hat{n} \), respectively.

(a) Show that \( e^{-\epsilon \hat{H}} = e^{-\epsilon \hat{H}_U} e^{-\epsilon \hat{H}_t} + O(\epsilon^2) \)

(b) Show that

\[ e^{-\epsilon \hat{H}_U} e^{-\epsilon \hat{H}_t} \Psi(\theta) = \sum_{\eta'} \int \frac{d\theta'}{2\pi} e^{i\eta' \theta_0 - \epsilon H_t(\eta') - \epsilon H_U(\eta')} \Psi(\theta'). \quad (1) \]

(c) Show that

\[ e^{-\epsilon T \hat{H}} \Psi(\theta) = \lim_{\epsilon \to 0} \sum_{n^{(1)}, n^{(2)}, \ldots, n^{(M)}} \int \frac{d\theta^{(1)}}{2\pi} \frac{d\theta^{(2)}}{2\pi} \ldots \frac{d\theta^{(M)}}{2\pi} e^{i \sum_{l=1}^{M} [\theta^{(l)} - \theta^{(l+1)}] - \epsilon H_t(\theta^{(l)}) - \epsilon H_U(\eta^{(l)})]} \Psi(\theta^{(1)}) \bigg|_{M=T/\epsilon}, \quad (2) \]

where \( \theta^{(M+1)} = \theta \).