Homework

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August 2022

1 Homework question: Unit converter

Convert (You should first determine the mass dimension of the units. For example, $Hz = s^{-1}$):

- pc to meter
- K to eV
- s to eV
- Tesla to eV
- V/m to Tesla
- meter to eV
- Solar mass in GeV
- GeV/cm^3 to eV

2 Homework question: Freeze in dark matter

A second general scenario for thermal dark matter production is known as freeze-in, where the universe starts with zero density of a particle X, and the density increases because Standard Model particles interact and turns into particle X as the universe cools down and expand.

Here, the example we will consider is dark photon, which acts very much like our photon, but it has a mass m_V and is coupled to us with a coupling e_V which is much smaller than the coupling of electromagnetism e ($e_V \ll 1$). The rate of production of this new field V from each Standard model particle, can be written, very cruedly, as

$$\Gamma \sim e_V^2 T \tag{1}$$

for temperature of the universe $T > m_V$ and 0 otherwise.

1. How does this rate compare to Hubble? When is production of this particle maximized?

2. At this temperature, roughly how many particles are produced per standard model particle?

3. In reality, the electrons decouple when $T < m_e$ so the production of V gets very suppressed once $T < m_e$. Assuming the production is around the time when $T \sim m_e$, how does the parametrics change as a result?

3 Axion density

Work out the details of the evolution of the axion dark matter density. If you are interested, read about how QCD axion dark matter is different.

4 Homework question: The powerful clock

Consider a dark matter candidate which couples like

$$\mathcal{L} \supset \frac{\phi}{F_\phi} F_{\mu\nu} F^{\mu\nu} \tag{2}$$

in addition of the Standard Model Lagrangian (You are allowed to be a $\pi = 3 = 1$ physicist like me in the questions that contain numbers).

1. How does the fine structure constant α change if a background ϕ field oscillates as $\phi(t) = A_\phi \cos m_\phi t$?

2. Now consider two atomic clocks made out of Sr and Yb neutral atoms, which has energy levels that depends on the fine structure constant differently. How does the the ratio depends on the fine structure constant.

3. Thanks to the incredible technological advances, such a ratio is measured to the precision of 10^{-18} for oscillations with periods of a month, what does it imply on the precision of measurement of the ratio $\phi(t)/F_\phi$ at the same frequency?

4. Given the local dark matter density of $0.45\,\mathrm{GeV/cm^3}$, what is the limit on the scale of F_ϕ, how does it compare to the Planck scale $m_{\mathrm{pl}} = 6 * 10^{18}\,\mathrm{GeV}$?