Aspects of HEP – Data Reconstruction

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After our particles have interacted with our detector, we have to “read out” that information, and format it in such a way that we can understand it. Generally, that entails some sort of transducer that converts the energy deposited in our detector elements and transforms it into electrical signals. It is also customary to digitise the signals for further data processing.

- **Local reconstruction** starts with the detector readout (e.g., channel #28 has 7 ADC counts), and transforms it into intelligible data — a “hit”. This is usually low-level information localised to one subsystem, e.g., for a tracker detector it represents not much more than “a charged particle passed through this sensor”.

- **Global reconstruction** starts with the hits and tries to group them at a higher-level. Connecting the hits in a silicon detector to reconstruct the trajectory of a charged particle, or clustering the hits in a scintillator to reconstruct the deposit of a photon, fall in this category.

- **Global event description** uses the high-level objects to describe the event as a whole, ready to be connected to a physics process interpretation. Connecting energy clusters to a track to try to reconstruct an electron or a charged hadron, or connecting tracks in two different tracker systems to reconstruct a muon, fall in this category.
Coordinate System

\[
\eta = \ln \left( \tan \left( \frac{\theta}{2} \right) \right)
\]

\[
\rho = \sqrt{x^2 + y^2}
\]
Local reconstruction is very dependent on the detector implementation details. We will use CMS’s crystal electromagnetic calorimeter (ECAL) as an example.

**ECAL**

- Around 80000 PbWO$_4$ (lead glass) scintillating crystals.
  - High density (8.3 g/cm$^3$), short radiation length (0.89 cm), small Molière radius (2.2 cm).
  - 80% of the light is emitted in 25 ns, $\sim$420 nm wavelength, 4.5 photoelectrons per MeV.
- Readout: avalanche photodiodes (APDs) and vacuum phototriodes (VPTs) for barrel ($|\eta| < 1.479$) and endcap ($1.479 < |\eta| < 3.0$) regions, respectively.
  - APDs exploit the photoelectric effect in an avalanche mode diode to convert light to electricity.
  - VPTs are single stage (cathode–anode–dynode) photomultipliers.
ECAL Geometry

Barrel ECAL (EB)

Preshower (ES)

Endcap ECAL (EE)
ECAL Readout Chip

- ECAL readout and precision performance requirements ⇒ front end signals digitised to better than 12 bits.
- Strategy: multiple gain ranges \((\times 12, \times 6, \times 1)\) to span the overall dynamic range.
  - Digitise and transmit the signals only for the highest unsaturated range.
  - 12-bit analog-to-digital converter (ADC) is now sufficient.
  - Parallel gain channels in a multi-gain pre-amplifier (MGPA), coupled to a multi-channel ADC.
  - "Channel-in-range" decision taken by digital logic following the conversion stages.

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Radiation damage leads to transparency loss!

![Graph showing ECAL response vs integrated luminosity]

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Hit reconstruction refers to the process of estimation of (energy, time, position) of an energy deposit in an ECAL crystal.

At a given time sampling: 12-bit word for ADC counts + bits to encode the gain ($\times 12$, $\times 6$, $\times 1$). For a given event we want to save, we take 10 time samples: 3 readings before the selected bunch-crossing and 7 readings afterwards. That information comprises a “digi”,

From an ECAL “digi”, we want to reconstruct four quantities: amplitude (the peak of the pulse shape), pedestal (the baseline), jitter (the time when the maximum of the pulse shape occurs) and $\chi^2$ of the fit (see next slides). After that we still want a global scale factor (ADC counts to GeV) and intercalibration constants.
The multifit method allows for better timing information on the pulse.

**Diagram:**
- CMS simulation, $\sqrt{s}=13$ TeV
- PU=20/BX, 25 ns
- Observed signal
- Total pulse
- In-time pulse
- Out-of-time pulses

**Graph:**
- ECAL BARREL, hit E > 1 GeV
- $\sqrt{s} = 13$ TeV, 25 ns
- Preliminary

- entries/0.2 ns
- hit time (ns)
Calibrate the Energy

- Use particles of very well known mass to calibrate the detector.
  - $\pi^0 \rightarrow \gamma\gamma$, mass of 135 MeV.
  - $Z \rightarrow ee$, mass of 91.1876 GeV.
- Exploit detector $\phi$-symmetry to intercalibrate crystals.
- Exploit $E/p \approx 1$ for high-energy electrons.

![Graph showing invariant mass distribution](image)
Track reconstruction refers to the process of using the hits, obtained from the local reconstruction, to obtain estimates for the momentum and position parameters of the charged particles responsible for the hits (tracks).

- Translate the hits’ local coordinate system to the tracks’ global coordinate system.
  - Misalignment of detector elements: additional uncertainty.

Traditionally divided in two steps:

- **Track finding:** decide, within the set of reconstructed hits \( \{X_i\} \), which subset \( \{X_i\}_k \) makes up a given track candidate \( k \).

- **Track fitting:** from the subset of hits that make up a given track candidate, estimate the parameters – production vertex position \( x_v \) and momentum at vertex \( p_v \) – of the originating particle.
Tracking: A Hard Problem
Motion in Magnetic Field

In a magnetic field the motion of a charged particle and finally is determined by the Lorentz Force

\[ \frac{dp}{dt} = e \mathbf{v} \times \mathbf{B} \]

Since magnetic forces do not change the energy of the particle

\[ m_o \gamma \frac{d\mathbf{v}}{dt} = e \mathbf{v} \times \mathbf{B} \]
\[ m_o \gamma \frac{d^2 \mathbf{r}}{dt^2} = e \frac{d\mathbf{r}}{dt} \times \mathbf{B} \]

using the path length \( s \) along the track instead of the time \( t \), \( ds = vdt \) we have

\[ m_o \gamma v \frac{d^2 \mathbf{r}}{ds^2} = e \frac{d\mathbf{r}}{ds} \times \mathbf{B} \]

\[ \frac{d^2 \mathbf{r}}{ds^2} = \frac{e}{p} \frac{d\mathbf{r}}{ds} \times \mathbf{B} \]

- Inhomogeneous \( \mathbf{B}(s) \): solve differential equation to find the trajectory \( r(s) \).
- Homogeneous \( \mathbf{B} \): helix trajectory – circle in plane transverse to \( \mathbf{B} \).
**Magnetic Spectrometer**

**Solenoidal field**

- cylindrical symmetry
- deflection in $x - y \ (\rho - \phi)$ plane
- tracking detectors arranged in cylindrical shells
- measurement of curved trajectories in $\rho - \phi$ planes at fixed $\rho$

**Dipole field**

- rectangular symmetry
- deflection in $y - z$ plane
- tracking detectors arranged in parallel planes
- measurement of curved trajectories in $y - z$ planes at fixed $z$
The Helix Equation

Parametric form:

\[
\begin{align*}
  x(s) &= x_o + R \left[ \cos \left( \Phi_o + \frac{h s \cos \lambda}{R} \right) - \cos \Phi_o \right] \\
  y(s) &= y_o + R \left[ \sin \left( \Phi_o + \frac{h s \cos \lambda}{R} \right) - \sin \Phi_o \right] \\
  z(s) &= z_o + s \sin \lambda 
\end{align*}
\]

\(x_o\) and \(y_o\) are the coordinates at \(s = 0\), while \(\Phi_o\) is related to the slope of the tangent to the circle at \(s = 0\).

In the transverse plane \(x - y\), we can relate the curvature radius \(R\) to the transverse momentum \(p_T\) as:

\[
R = \frac{p_T}{0.003 q B}
\]

\(R\) in cm, \(p_T\) in GeV, \(q\) in elementary charge, \(B\) in T.

\(\lambda\) is the dip angle. \(h = \pm 1\) is the sense of rotation on the helix. The projection on the \(x - y\) plane is a circle:

\[
(x - x_o + R \cos \Phi_o)^2 + (y - y_o + R \sin \Phi_o)^2 = R^2
\]
Methods of Track Finding

Conformal Mapping

Circle going through the origin: \((x - a)^2 + (y - b)^2 = R^2 = a^2 + b^2\). Define:

\[
\begin{align*}
u &= \frac{x}{x^2 + y^2}, \\
v &= \frac{y}{x^2 + y^2},
\end{align*}
\]

leading to

\[
v = \frac{1}{2b} - u \frac{a}{b}
\]

How to obtain track candidates:

- Convert hits from \((x,y)\) to \((u,v)\).
- In the \((u,v)\) coordinate system, the larger the momentum, the closer line is to origin.
- Approximate line as going through origin – hits of a track have constant \(\phi_v\).

In practice, we search for peaks in the histogram of \(\phi_v\) variable of all hits.
All the blue points have, approximately, the same $\phi_v$. 
Methods of Track Finding

**Hough Transform**

Remember: the radius of curvature is \( R = \frac{p_T}{0.003 qB} \). We are interested in particles originating at or close to the interaction point. The trajectory of such particles in the transverse plane is described by the following equation:

\[
\frac{r}{2R} = \sin(\varphi - \phi) \approx \varphi - \phi
\]

Here \( \phi \) is the angle of the track in the transverse plane at the origin \( (\phi = \Phi_o^{\text{helix}} - \pi/2) \). The small angle approximation is valid for tracks with \( p_T \gtrsim 2 \text{ GeV} \). Combine the two equations:

\[
\phi = \varphi - \frac{0.0015 qB}{p_T} \cdot r
\]

So, one hit in \((r, \varphi)\) maps to a straight line in the track parameter space \((q/p_T, \phi)\).
If several hits are produced by the same particle, all the lines intersect at the same point.

- Locate the intersection to identify a track.
- Intersection coordinates locate provide track parameters \((q/p_T, \phi)\).
- **Caveat:** the gradient of each line is proportional to the hit radius \(r\) – always positive. Better use variables \(r_T, \phi_T\) instead:

\[
r_T = r - T, \quad \phi_T = \phi_{\text{track}}(r_T)
\]

Now \(r_T\) can have both signs; this leads to a larger range of line gradients and more precision in the measurement of the intersection point.
Methods of Track Finding

Track Road and Track Following

These are local approaches, i.e., start already from a subset of the hits.

- **Track Road:** start from hits that could have been created by the same particle. Use the expected trajectory shape to interpolate amongst the hits, creating a ”road” around the trajectory. All hits that fall in the road constitute the track candidate.

- **Track Following:** start from a ”track seed” – a subset of hits that for some reason are designed to be more accurate. From the seed, extrapolate until the next detector element containing a hit. The closest hit becomes part of the track candidate. Iterate until either 1) too many layers have missing hits or 2) the end of the detector system is reached.
Elements of Track Fitting

- Track parameterization: depends on detector geometry. For helix trajectory in solenoid, we could use the parameter set \( \{x_o, y_o, \Phi_o, \lambda, R\} \) we discussed.

- Track model: how the track parameter or state vector \( q \) at a given surface \( k \) depends on the state vector on a different surface \( i \): \( q_k = G_{k|i}(q_i) \), where \( G_{k|i} \) is the track propagator from surface \( i \) to \( j \).

- Error propagation: during the track parameter estimation procedure, propagation of the track parameter covariance matrix along with the track parameters themselves is often requested.

- Material effects: ionization energy loss, multiple Coulomb scattering, bremsstrahlung, all affect the particle propagation.

- Measurement model: \( h_k \) describes the functional dependence of the measured quantities in layer \( k \), \( m_k \), on the state vector at the same layer: \( m_k = h_k(q_k) \).
Track Fitting – Momentum Measurement (1)

Particle momentum projected along two directions:

In the $\rho - \phi$ plane we measure the transverse momentum $p_T$ through the curvature radius $R$:

\[
p_T = p \cos \lambda = 0.003Br
\]

In the $\rho - z$ plane we measure the dip angle $\lambda$:

Orders of magnitude:

\[
\begin{align*}
    p_T &= 1 \text{ GeV} \quad B = 2T \quad R = 1.67 \text{ m} \\
    p_T &= 10 \text{ GeV} \quad B = 2T \quad R = 16.7 \text{ m}
\end{align*}
\]

The sagitta $s$:

\[
R^2 = L^2 + (R - s)^2\]

\[
R = \frac{s}{2} + \frac{L^2}{2s}\]

\[
s \ll R \Rightarrow R \approx \frac{L^2}{2s}\]

Assuming a track length of 1m:

\[
\begin{align*}
    p_T &= 1 \text{ GeV} \quad s = 7.4 \text{ cm} \\
    p_T &= 10 \text{ GeV} \quad s = 0.74 \text{ cm}
\end{align*}
\]
Once we have measured the transverse momentum and the dip angle the total momentum is

\[ p = \frac{p_T}{\cos \lambda} = \frac{0.003 BR}{\cos \lambda} \]

The error in the momentum is easily calculated

\[ \frac{\partial p}{\partial R} = \frac{p_T}{R} \]

\[ \frac{\partial p}{\partial \lambda} = -p_T \tan \lambda \]

\[ \left( \frac{\Delta p}{p} \right)^2 = \left( \frac{\Delta R}{R} \right)^2 + (\tan \lambda \Delta \lambda)^2 \]

We need to study:

- the error on the radius measured in the bending plane \( \rho - \phi \);
- the error on the dip angle in the \( \rho - z \) plane;
- contribution of multiple scattering to momentum resolution.

In an hadronic collider the main emphasis is on the transverse momentum \( p_T \). Since the elementary processes happen amongst partons that are not at rest in the lab frame, we employ momentum conservation only in the transverse plane.
To start the Kalman Filter we need a “seed”;
the position on the next plane is predicted;
the measurement is considered;
prediction and measurement are merged (filtered);
iterate: prediction – measurement – filtering – ...

Filtering: weighted average of the new measurement $y_n$ and the new prediction $y_p$.
Smoothing: refine each position estimate with information from all measurements.
Kalman Filter – The Equations

Dimensions

- $P$ parameters for track model.
- $D$ measurements $m_k$ at each point (usually $D < P$).
- $K$ measurement points ($N = K \times D$).
- State of the system at index $k$ is $q_k$.

Starting point

- Initial set of parameters (seeds): first measurements.

Iterative method

- Using system equation $q_k = G \cdot q_{k-1} + \omega_k$.
  - $G = P \times P$ matrix, $\omega$ = perturbation associated with covariance $P \times P$ matrix $V_\omega$.
- Add new point, update parameters and cov., using meas. equation $m_k = H \cdot q_k + \epsilon_k$.
  - $H = D \times P$ matrix, $\epsilon$ = measurement error associated with diagonal cov. $D \times D$ matrix $V_m$.
  - Weighted means of prediction and measurement using variance $\Rightarrow \chi^2$ fit.

$$q_k = \left( V_{k|k-1}^{-1} q_{k|k-1} + H^T V_{m_k}^{-1} m_k \right) \cdot \left( V_{k|k-1}^{-1} + H^T V_{m_k}^{-1} H \right)^{-1}$$
All of the subsystems provide basic elements: silicon tracks, calorimeter hits (EM and hadronic), muon spectrometer tracks, ...

Basic idea: correlate basic elements to identify all final-state particles. Combine corresponding measurements for ultimate accuracy in particle properties.

- Initially developed and used by ALEPH (LEP).
- Had not been used with much success in hadron colliders – detector granularity was not fine enough!

Complicated algorithm that implements a set of links:

- Electrons leave both tracks and EM calorimeter deposits; photons leave only the latter.
- Muons leave tracks both in silicon trackers and in muon spectrometers.
- Charged hadrons leave both tracks and hadronic calorimeter deposits; neutral hadrons leave only the latter.
- Electrons emit photons during bremsstrahlung, all that energy must be reclustered.
- A hadronic $\tau$ decay produces just a few hadrons and one neutrino.

The output of the PF algorithm is a list of final-state reconstructed particles.
Example: Electron Reconstruction with ECAL + Tracker
Hadronic Jets and Missing $p_T$

We saw in the first lecture that quarks and gluons fragment and hadronize, producing a collimated, high-momentum spray of hadrons flying roughly in the same direction – a hadronic jet. A jet algorithm is an iterative, tuneable procedure that clusters close particles and creates the jet objects.

In the same vein, we can cluster all observed particles. This allows for an important cross-check: the total transverse momentum in a hadron collision should be zero. If, after the overall clustering, a sizeable transverse momentum remains, that would be evidence for an undetected particle being produced!

We define the missing transverse momentum $p_T^{\text{miss}}$:

$$p_T^{\text{miss}} = - \sum_i p_T^i$$
Event Displays of Calorimeter Jets

ATLAS event display for a highly asymmetric heavy-ion dijet event, with only one reconstructed jet at a transverse energy $ET > 100$ GeV.

Jet 1,
- $pt = 2.88$ TeV
- $\eta = -0.364$
- $\phi = 1.915$

Jet 0,
- $pt = 3.04$ TeV
- $\eta = 0.059$
- $\phi = -1.235$

CMS Experiment at LHC, CERN
Data recorded: Mon Oct 12 2015 EEST
Run/Event: 258749 / 549854773
Lumi section: 355
Dijet Mass: 6.14 TeV
Event Display of a PF Hadronic Jet
Online Reconstruction

- With a collision rate close to 40 MHz, impossible to record and store all collisions happening at the LHC experiments.
- **Trigger system** implemented to keep only the more interesting collisions – quasi-real time reconstruction.
- Level 1 (L1) trigger in CMS:
  - Customizable hardware (ASICs and FPGAs).
  - Output rate: 100 kHz (detector readout constraint).
  - Timing: 4 \( \mu s \) (available buffer).
  - Coarse granularity.
- High Level Trigger (HLT) in CMS:
  - PC farm with light version of the offline software.
  - Output rate: 1 kHz average over LHC fill.
  - Timing: \( \lesssim 380 \, \mu s \).
  - Full detector granularity.
A Fully Reconstructed Event

![Diagram of a fully reconstructed event](image)
Reconstruction is ready…

… onwards to Data Analysis!