EFT – problems 1

Question 1

Consider the multipole expansion of a potential $\phi(\vec{r})$ of a charge distribution $\rho(\vec{r})$ as discussed in the lecture. Suppose

$$\rho(\vec{r}) = Q\left(\frac{1}{32\pi r_0^5}\right) r^2 e^{-r/r_0} \cos^2\theta \tag{1}$$

where $r = |\vec{r}|$ and r_0 is a constant. Find the explicit expressions for the monopole and dipole contributions to $\phi(\vec{r})$. For what distances (expressed in terms of r_0) from the center of the distribution one can expect the monopole giving a good representation for $\phi(\vec{r})$?

Question 2

Work out the $\mathcal{O}(1/M^6)$ (on-shell) contribution to the scattering amplitude $A(p_1, p_2, p_3, p_4)$ discussed in the lecture.