How to evaluate such integrals?

* easy when \( S[\Psi] \) is quadratic in \( \Psi \), e.g.

\[
\frac{1}{\mathcal{Z}} \int d\Psi \Psi(x_1) \Psi(x_2) \Psi(x_3) \Psi(x_4) e^{i S[\Psi] + i \int d^4 x J(x) \Psi(x)} = \frac{1}{\mathcal{Z}} \cdot \frac{\delta}{(i)^4 \delta J(x_1) \delta J(x_2) \delta J(x_3) \delta J(x_4)} e^{i W[J]}
\]

\[
\frac{\delta}{(i)^4 \delta J(x_1) \delta J(x_2) \delta J(x_3) \delta J(x_4)} e^{i W[J]} = \frac{1}{\mathcal{Z}} \cdot \frac{\delta}{\delta J(x_1) \delta J(x_2) \delta J(x_3) \delta J(x_4)} e^{i W[J]} \quad \text{(139)}
\]

\[
D(x_1 - x_2) D(x_3 - x_4) + D(x_1 - x_3) D(x_2 - x_4) + D(x_1 - x_4) D(x_2 - x_3) \quad \text{(140)}
\]

\[\uparrow \text{Wick's theorem}\]

* \( S[\Psi] \) is not quadratic \( \rightarrow \) interactions

- when applicable, use perturbation theory

\[
S[\Psi] = S_0[\Psi] + S_{\text{int}}[\Psi] \quad \text{(141)}
\]

\[\uparrow \text{quadratic in } \Psi \] \[\uparrow \text{non quadratic} \]
\[
\begin{align*}
&\text{Therefore, a } n \text{-point correlation function can be evaluated as follows:}\nonumber \\
&C(x_1, \ldots, x_n) = \frac{1}{Z} \int \Delta \psi(x_1) \cdots \psi(x_n) e^{i S[\psi]} \nonumber \\
&= \frac{1}{Z} \int \Delta \psi(x_1) \cdots \psi(x_n) e^{i S_0[\psi]} \nonumber \\
&\quad \quad + \frac{1}{Z} \int \Delta \psi(x_1) \cdots \psi(x_n) \left( i S_0[\psi] \right)^2 \left( i S_0[\psi] \right)^3 \cdots 1 \nonumber \\
&\quad \quad + \cdots \quad \text{Note: have to expand the denominator } Z \text{ as well} \nonumber \\
&= C^{(2)}(x_1, \ldots, x_n) + C^{(3)}(x_1, \ldots, x_n) + \cdots \nonumber \\
&\quad \quad \text{E.g., (140)}
\end{align*}
\]
Suppose that the interaction term \( S_{\text{int}}[\phi] \) is

\[
S_{\text{int}}[\phi] = \int d^4 x \text{Lint}(\phi), \quad \text{Lint} = -\frac{\lambda}{4!} \lambda \left( \phi(x) \right)^4
\]

1st order term: \( C(x_1, \ldots, x_N) \)

\[
\left[ \frac{\partial}{\partial \phi(x_1)} \cdots \frac{\partial}{\partial \phi(x_N)} \right] \left[ -\frac{\lambda}{4!} \left( \phi(x) \right)^4 \right] e^{i S_{\text{int}}[\phi]} =
\]

\[
= -\frac{1}{4!} \lambda \int d^4 x \int \frac{\partial}{\partial \phi(x_1)} \cdots \frac{\partial}{\partial \phi(x_N)} \left( \phi(x) \right)^4 e^{i S_{\text{int}}[\phi]}
\]

we Wick's theorem

sum over all pairwise contractions

\( N \) even \( \rightarrow \) OK

\( N \) odd \( \rightarrow \) zero, in absence of SSB

Note, however: in the contractions, there are terms in which \( \phi(x_1) \cdots \phi(x_N) \) are not contracted with the \( \phi(x) \) \( \phi(x) \) \( \phi(x) \)

\( \Rightarrow \) they are "disconnected"
Example: \( N = 4 \)

\[
\Phi(x_1) \Phi(x_2) \Phi(x_3) \Phi(x_4) \Phi(x) \Phi(x) \Phi(x) \\
\rightarrow D(x_1 - x) \ D(x_2 - x) \ D(x_3 - x) \ D(x_4 - x) \\
+ \cdots + \left[ D(x_1 - x_2) \ D(x_2 - x_4) + \cdots \right] \ D(x-x) \ D(x-x)
\]

1st line: connected

\[
\begin{array}{c}
\text{x}_1 \ \
\text{x}_2 \ \
\text{x}_3 \ \
\text{x}_4
\end{array}
\]

2nd line: disconnected

\[
\begin{array}{c}
\text{x}_1 \ \
\text{x}_2 \ \
\text{x}_3 \ \
\text{x}_4
\end{array}
\]

**BUT:** the disconnected cancel when expanding the \( \frac{1}{Z} \) in powers of \( \lambda \)

- the cancellation happens order-by-order

\[
\frac{1}{Z} \left( \Phi(x_1) \Phi(x_2) \Phi(x_3) \Phi(x_4) \right) e \leq \text{only connected terms contribute}
\]

\( - - - - - - (143) \)
Scattering amplitude: 2 particles → 2 particles

\[ A(k_1, p_2; p_3, p_4) \to \langle \Phi_3 | \Phi_4 | \Phi_1, \Phi_2 \rangle \]

Theorem: will not prove it, see e.g. Huke's lectures

\[ L.S. = \text{reduction formula:} \]

\[ i A(k_1, p_2; p_3, p_4) (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) = C(k_1, k_2, p_3, p_4) \quad \text{on-shell} \]

\[ \text{where } C(k_1, \ldots, p_4) \text{ is the Fourier transform} \]

\[ C(k_1, \ldots, p_4) = \int d^4 x_1 \ldots d^4 x_4 \ e^{-i(k_1 \cdot x_1 + k_2 \cdot x_2 - p_3 \cdot x_3 - p_4 \cdot x_4)} \ C(x_1, \ldots, x_4) \]

with \( C(x_1, \ldots, x_4) \) being the 4-point correlation function:

\[ \text{amb. in Eq. (144) means amputation of external propagators} \]

\[ \text{amb. in Eq. (144) means amputation of external propagators} \]

\[ \text{amb. in Eq. (144) means amputation of external propagators} \]
on-shell in Eq. (144) means that all external particle on-shell: \( p_i^2 = m_i^2 \)

Exercise: show that for \( \lim t = -\frac{1}{4!} \lambda \phi(x) \):

\[
A(k_1, \ldots, k_4) = -\lambda
\]

to first order in \( \lambda \).
Construction of an EFT

1. Determine the relevant degrees of freedom
   - not always easy, e.g., physics at the confinement scale in QCD

   What fields?

2. Identify the symmetries
   - also not always easy, e.g., emergent symmetries due SSB (smallness of pion masses)

   What interactions?

3. Find an expansion parameter and get an first order description

   What power counting to use

Ready to write down the Lagrangian of your EFT

Construction of the Standard Model:

- essentially followed * and **
- not much *** - but extremely important in EFT
Key principle: to describe physics at scale $m^2$, do not need to know field content, symmetries.

Dynamics at scale $M^2 \gg m^2$

Generically: (scalar field only)

The lagrangian is written as

$$\mathcal{L} = \sum_n C_n(\lambda) \, O^{(n)}(\phi, \partial \phi) \quad \text{--- (147)}$$

where $O^{(n)}$ is polynomial in $\phi$ and $\partial \phi$

$n$: mass dimension of $O^{(n)}$ \quad [\text{mass}]^n

$\lambda$: cutoff, large mass scale

Mass dimension of a field: $h = 1$, $C = 1$

$\lambda C \sim 200 \text{ MeV fm}^4$, \quad [\text{mass}] = \text{MeV or fm}^{-1}$

Dimension of the action

$$\frac{i}{\hbar} S \rightarrow [S] = [h] = [1] \rightarrow (\text{mass})^0 \quad \text{--- (148)}$$
Suppose a Lagrangian density of the form

\[ L = \frac{1}{2} \left( \partial \Phi \right)^2 - \frac{1}{2} m^2 \Phi + \chi \Phi^4 + \frac{g}{6!} \Phi^6 \]  

\[ \cdots \cdots \ (149) \]

* Mass dimension of \( L \):

\[ S = \int d^4x \ L \rightarrow d^4x \rightarrow L = \eta^4 \left[ d^4x \right] = -4 \]

\[ \Rightarrow \left[ L \right] = 4 < \eta \]  

\[ \cdots \cdots \ (150) \]

* Mass dimension of \( \Phi \):

\[ \int d^4x \ m^2 \Phi^2 \]

\[ (\text{mass})^0 = (\text{mass})^{-4} (\text{mass})^2 \left[ \Phi \right]^2 \]

\[ \left[ \Phi \right] = 1 \leftrightarrow \eta \]  

\[ \cdots \cdots \ (151) \]

* Mass dimension of \( \partial \Phi \):

\[ \frac{1}{\partial x} \Phi \rightarrow \frac{1}{L} \Theta \rightarrow \left[ \partial \Phi \right] = 2/L \]  

\[ \cdots \cdots \ (152) \]

\[ \text{LEFT} = A_0 + A_1 \Phi + A_2 \Phi^2 + A_3 \Phi^3 + A_4 \Phi^4 \\
+ A_6 \Phi^6 + D_1 \left( \partial \Phi \right)^2 + D_2 \left( \partial \Phi \right)^2 + D_3 \Phi \left( \partial \Phi \right) \\
+ D_4 \Phi^2 \left( \partial \Phi \right)^2 + \cdots \]

\[ \text{Lorentz} \]  

\[ \cdots \cdots \ (153) \]
\[ [A_0] = 4 \implies A_0 = \bar{A}_0 \Lambda^4 \]
\[ [A_2] = 2 \implies A_2 = \Lambda^2 \bar{A}_2 \]
\[ [A_3] = 1 \implies \text{break} \ G - 3 - G \text{ symmetry} \]
\[ \text{If symmetry not required, it is allowed, e.g., } \psi, \psi^\dagger \ldots \]
\[ [A_4] = 0 \implies A_4 = \Lambda/4! \]
\[ [A_6] = -2 \implies A_6 = \bar{A}_6 / \Lambda^2 \]
\[ [D_2] = 4 \implies D_2 = 1 \implies \text{correct kinetic energy} \]
\[ [D_4] = 6 \implies D_4 = \bar{D}_4 / \Lambda^2 \]

\[
\text{One expects factors } \frac{\Lambda}{\Lambda^k} \quad k > 0 \text{ expected to be suppressed => less important.}
\]

We are back to the 1st lecture.

\[
\text{Literature:}
\]

1) Zee: Quantum field theory on the nutshell

2) Kaplan: Effective field theories, arXiv: 9503035

3) Burgess: An introduction to effective field theory

4) Luke: Quantum field theory I - Lecture notes
   www.physics.utoronto.ca/~luke/PHY2403F/References/lecturenotes.pdf
Stewart: Effective Field Theory, MIT Open Courseware
www.ocw.mit.edu search here