Effective Field Theory - EFT

1) Motivation

1.1) Meaning

An EFT is a field theory for describing the dynamics of a system at length scales large compared to a given cutoff.

Long wavelengths are effectively decoupled from short wavelengths

There is a natural separation of scales in the problem.

Statements can be reformulated in terms of energy scales → EFT, a theory for the dynamics at energies smaller compared to a given cutoff energy.

Two examples in classical physics:
1.2) Gravity close to Earth's surface

\[ F = m g \]  \quad (1)

- get this from the "fundamental" Newton's law

\[ F = \frac{G M m}{R^2} \]  \quad (2)

For \( h \ll R_e \):

\[ F = \frac{G M m}{R_e^2 (1 + \frac{h}{R_e})^2} \approx \frac{G M m}{R_e^2} \left( 1 - 2 \frac{h}{R_e} + \frac{3}{4} \frac{h^2}{R_e^2} + \cdots \right) \]  \quad (3)

\[ h = 10 \text{m}, \quad R_e \approx 6371 \text{km} < \text{equator} \]

\[ m g \left( 1 - 0.000000031 + 0.0000000000074 + \cdots \right) \]

When this level of precision is OK

Forget about Newton's law

Larger \( h \): need Newton's law, e.g. for satellites

For GPS, Newton's law is not enough, need GR

1.3) Multipole expansion: charge distribution \( \rho(\vec{r}) \)

\[ \phi(\vec{r}) = \frac{1}{4\pi} \int d^3\vec{r}' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} \]  \quad (4)
- Separation of scales \( <r'>= \int d^3r' \rho(r') \) 
\[<r'> \ll r \] \hspace{1cm} \text{(5)}

- Expansion of integrand in powers of \( \frac{r'}{r} \)

\[
\frac{1}{|r' - r|^l} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{4\pi}{2l+1} \frac{r}{r' + \lambda} Y_{lm}(\theta', \phi') Y_{lm}(\theta, \phi) \]
\hspace{1cm} \text{(6)}

Using this in (4) for \( r > r' \): \( r \geq r' \), \( r' < r \)

\[
\phi(r') = \sum_{l,m} \frac{4\pi}{2l+1} \frac{Y_{lm}(\theta, \phi)}{r'^{l+1}} Q_{lm} \]
\hspace{1cm} \text{(7)}

\[
Q_{lm} = \int d^3r' \rho(r') r' l^k Y_{lm}(\theta', \phi') \]
\hspace{1cm} \text{(8)}

\[
\phi(r) = \frac{Q}{r} + \frac{d}{r^2} + \ldots \]
\hspace{1cm} \text{(9)}

\[
Q = \int d^3r \rho(r) : \text{point charge} \] \hspace{1cm} \text{(10)}

\[
d = \frac{4\pi}{3} \sum_{l=0}^{+1} \frac{Y_{lm}(\theta, \phi)}{r' l^k} \int d^3r' \rho(r') Y_{lm}^*(\theta', \phi') \]
\hspace{1cm} \text{(11)}

\( \rho \text{ dipole} \)

The closer one gets to the charge distribution, more structure one sees.\( r \ll <r' > \), breakdown scale
1.4) Aim of this course: at the end you should be able to understand (and reproduce) the following example of two spinless particles, one is much lighter than the other: fields \( \phi(x) \) and \( \overline{\phi}(x) \) masses \( m \) and \( M \), with \( m \ll M \) [Kaplan, Burgess]

Classical action:

\[
S_c = \int d^4x \left[ \frac{1}{2} \partial^\mu \phi \partial^\mu \phi + \frac{1}{2} \partial^\mu \overline{\phi} \partial^\mu \overline{\phi} - V(\phi, \overline{\phi}) \right] \tag{12}
\]

\[
V(\phi, \overline{\phi}) = \frac{1}{2} m^2 \phi^2 + \frac{1}{2} M^2 \overline{\phi}^2 + \frac{1}{4!} g \phi^4 + \frac{1}{2} \lambda \phi^2 \overline{\phi}^2 + \ldots \tag{13}
\]

Consider \( \phi - \phi \) scattering with c.m. energies \( E_{\text{cm}} \ll M \)

\[
\begin{align*}
\phi + \phi & \rightarrow \text{lowest order in perturbation theory} \\
\phi + \phi & \rightarrow \text{flatter form} \quad \phi^4 + \frac{1}{2} \phi^2 \overline{\phi}^2
\end{align*}
\]

\[
S\text{-matrix element:}
\]

\[
S(p_1, b_2, b_3, b_4) = \frac{i}{2\pi^3} \delta(p_1 + p_2 - b_3 - b_4) A(p_1, b_2, b_3, b_4) \tag{14}
\]
\[ \mathcal{A}(p_1, p_2, p_3, p_4) = \frac{1}{g^2} \left[ \frac{1}{(k_1-k_3)^2 - M^2} + \frac{1}{(k_1-k_4)^2 - M^2} \right] + \frac{1}{g^2 (k_1+k_2)^2 - M^2} \]

\[ \approx -g + \frac{\lambda^2}{M^2} \left\{ 3 + \frac{1}{M^2} \left[ \frac{(p_1-p_3)^2}{M^2} + \frac{(p_1-p_4)^2}{M^2} + \frac{(p_1+p_2)^2}{M^2} \right] \right\} \]

\[ = 6M^2 + 2p_1 \cdot (p_2-p_3-p_4) \]

\[ = 6M^2 - 2p_1^2 = 4M^2 \]

\[ \approx -g^2 + \frac{3\lambda^2}{M^2} + \frac{4\lambda^2 M^2}{M^4} \]

--- (15)

**Observations:**

1) Calculation of \( \mathcal{A} \) simplifies when one can neglect powers of \( \frac{m}{M}, \frac{\lambda}{M} \), etc.

2) Eq. (15) can be obtained from a theory involving only the \( c \) field:

\[ \text{Ve}_{(c)} = \frac{1}{4} \left( g - 3\frac{\lambda^2}{M^2} - 4\frac{\lambda^2 m^2}{M^4} \right) \Phi^4 \]

--- (16)
Knowing that some result is obtained when using (16) and (17) when \( \frac{m}{M} \ll 1 \); makes life easier instead of expanding full result. Guidance to what observables at low energies are more sensitive to properties of heavy particles.

Basic property of a EFT:

— when a large hierarchy of masses exists, a low energy description in terms of an EFT seems to be always possible.

1.5) How to construct an EFT? Be until Friday, you will know!
Categories of EFTs: Two ways they are used

- Top-down: High-energy theory is known

But we would like to have a simpler theory at low energies

- "Integrate out" heavy particles, and

- Match onto a low energy theory
  - find new operators & new low energy constants, e.g. previous example

\[ L \rightarrow \text{Low} = \sum L^{(n)} \text{Low} \text{-- expansion in decreasing importance} \]

\[ \text{describes more than } \]

but they coincide in the infrared and differ in the ultraviolet
- Bottom-up: high energy theory is not known, or it is known but matching cannot be done when integrating out degrees of freedom. Too difficult.

- Construct $\sum \mathcal{L}_n$ using most general operators/interactions consistent with symmetries.

- Couplings are unknown - determine from experiment or from full theory, e.g., lattice QCD.

- Precision desired - what $n$ to stop?

  Examples: 1) Superconductivity - see e.g., Kaplan

  2) QCD: Chiral perturbation theory

  3) Einstein gravity - quantum ed operators consistent with QFT