Aspects of Superconductivity - Exercises

1. **Landau Free Energy for FM with magnetic field:** For the Ising FM, we can write the free energy in presence of a magnetic field by adding an extra term which couples the magnetic field h to the order parameter Ψ, here the magnetization:

$$F_L[\Psi, h] = \frac{a}{2}\Psi^2 + \frac{b}{4}\Psi^4 - h\Psi \tag{1}$$

Minimize the energy in presence of the external field. Where are the minima for $T > T_c$ and $T < T_c$? Make the pictures for F_L as a function of Ψ for both cases. What you find is related to the idea of "explicit symmetry breaking" since now the external field selects a preferential orientation.

2. **Ginzburg-Landau Theory and Correlation Length:** Given the GL equation for a superconductor (now the order parameter is a complex number) in absence of fields:

$$f_{GL}[\Psi, \nabla\Psi] = \frac{\hbar^2}{2m}|\nabla\Psi|^2 + \frac{a}{2}|\Psi|^2 + \frac{b}{4}|\Psi|^4 \tag{2}$$

Minimize the GL free energy with respect to Ψ, and rescale the order parameter as $\Psi = \sqrt{a/b}f$, such that the differential equation you find looks more friendly and you can directly identify the coherence lenght $\xi_c = \sqrt{\hbar^2/2ma}$.

Show that the solution for this differential equation is $f(x) = \mathrm{Tanh}(x/(\sqrt{2}\xi_c))$. Make a figure of the behaviour of the order parameter accross a normal/superconductor interface.

3. **Minimal Coupling:** Show that within the Hamiltonian formalism, the substitution of $\mathbf{p} \to \mathbf{p} - q\mathbf{A}$ in the Hamiltonian for a free particle allows you to recover the correct equation of motion of a charged particle in presence of a magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$.

$$m\mathbf{a} = q\mathbf{v} \times \mathbf{B}. \tag{3}$$

4. **GL Equations:** Given the GL functional in presence of a magnetic field:

$$f_{GL}[\Psi, \nabla\Psi] = \frac{\hbar^2}{2m}|(\nabla - iq\mathbf{A}/\hbar)\Psi|^2 + \frac{a}{2}|\Psi|^2 + \frac{b}{4}|\Psi|^4 - \frac{\mathbf{B}^2}{2\mu_0}, \tag{4}$$

minimize with respect to Ψ and $\mathbf{A}$, and obtain the two GL Equations:

$$\mu_0\mathbf{j} = \frac{ie\hbar}{2m}(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi) - \frac{e|\Psi|^2\mathbf{A}}{m} \tag{5}$$

and

$$\frac{\hbar^2}{2m}\left[\frac{2ie\mathbf{A}\cdot\nabla\Psi^*}{\hbar}+\nabla^2\Psi^*-\frac{e^2\mathbf{A}^2}{\hbar}\Psi^*\right]-a\Psi^*-b|\Psi|^2\Psi^*=0 \tag{6}$$